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# Application of Convex Optimization Results of DE FINETTI's Problem for Proportional Reinsurance (A Case Study of Caarama Insurance Company in Algiers)

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**Abstract:**

This research aims to find the optimal retention level for a proportional reinsurance treaty based on the results of the convex optimization developed in De Finetti's model. The latter makes it possible to determine the level of retention that achieves the expected profit by the insurer while minimizing claims volatility. The convex functions appear abundantly in economics and finance. They have remarkable specificities that allow actuaries to minimize financial risks to which some institutions are exposed, especially insurance companies. Therefore, using mathematical tools to manage various risks is paramount. To remedy the optimization problem, we have combined the probability of failure method with the (De Finetti) model for proportional reinsurance, which proposed a retention optimization process that minimizes claim volatility for a fixed expected profit based on the results of the non-linear optimization.

**Keywords**: Convex Function, Nonlinear Optimization, Proportional Reinsurance, Risk Minimization, CAARAMA Insurance Company.
**JEL Codes** : C02, C25, C61, G22.

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## 1. Introduction

The insurer is responsible for guaranteeing the various risks to which its customers are exposed and for indemnifying claims that have occurred. However, the risks covered can strongly influence the insurer's security and possibly its result, encouraging it to seek coverage. As a business, the insurance company applies risk management techniques, namely reinsurance. Reinsurance allows the insurer to transfer part of its risks for a premium. It is an essential operation that guarantees the insurability of these risks and thus contributes to the development of insurance for damage and life insurance (Tosetti, Béhar, Fromenteau, & Ménart, 2011, p. 11).

Relations between insurers and reinsurers are based on a community of interest, each party seeking to determine the commitment that meets its objectives by setting the share of the risk to be retained. This part represents its retention, which is one of the most sensitive parameters in reinsurance (Blondeau & Partrat, 2003, pp. 14-83). As transactions between the two organizations are expected, a poor appreciation of the retention level directly impacts their profitability. To this end, each party must establish a reinsurance program that optimizes its commitment. The insurer's search for an optimal reinsurance structure is considered a major concern. Therefore actuaries have used convex functions. In finance, optimization algorithms are essential to define models that minimize risks. For this purpose, convex functions are the primary tool used in risk management (Charpentier, 2010). For example, De Finetti'sFinetti's work on nonlinear optimization has proved that convex optimization defines a dynamic approach to programming in insurance when determining optimal retention. Afterwards, mathematicians Glineur and Within extended De Finetti'sFinetti's results on other reinsurance treaties.

This paper aims to define an optimal proportional reinsurance structure based on De Finetti'sFinetti's convex optimization results to define an efficient reinsurance structure. To support this issue, we will develop a practical case devoted to determining an optimal retention level according to the De Finetti model using portfolio data from the life insurance company, followed by a comparative analysis with the retention level practised by the managers of the company. The analysis of this problem will also allow us to measure the effects of reassurance on the probability of ruin. The latter led us to define an efficient reinsurance structure for the combinations of profitability measures and risk.

## 2. The Reinsurance approach

### 2.1. Definition of reinsurance

Reinsurance can be defined as the technique by which an insurer transfers all or part of the risks it has taken out to another company. The idea that drives the reinsurance relationship is that of a sort of sharing of the ceding company by the reinsurer. The reinsurance transaction is based on the parties' good faith and generally involves a lasting partnership over time (Blondeau & Partrat, 2003, p. 2).

### 2.2. Forms of reinsurance

#### 2.2.1 Proportional reinsurance

We talk about proportional reinsurance when the reinsurer commits the same way to premiums and claims. There are again two types of treaties:

- The quota share treaty: This is the best-known treaty where the ceding company, which is the company that wants to reinsure, cedes to its reinsurer a percentage of the risk it has taken on as a direct insurer. This treaty form makes it possible to transfer a share of the claims and the same share of the premiums. In addition, the treaty provides that the reinsurer also bears a share of the management and acquisition costs corresponding in principle to the proportion of its commitment to the portfolio. This treaty is characterized by equality between the proportion of premiums received by the reinsurer and the proportion of the cost of claims transferred to the reinsurer.
- The treaty packed or treated more than capital (surplus): To limit the drawbacks mentioned with the quota share treaty, we can introduce the full surplus treaty, which seeks to homogenize the net portfolio by keeping all of the small risks and by limiting the custody on the most critical risks to a level deemed acceptable: whole. Once the full has been determined, the ceding company retains all the risks with exposures lower than the full. The risks exceeding the full are distributed between the ceding company and the reinsurer.

#### 2.2.2. Non-proportional reinsurance

Unlike proportional reinsurance based on the original commitment and proportional cession, the number of claims and the limited scope of coverage predominates in non-proportional reinsurance.

In the context of non-proportional reinsurance coverage, the following treaties can be cited:

- The excess of loss treaty: Intuitively, we can estimate that the actual losses primarily influence the volatility of the loss experience. To reduce the volatility of claims, it is preferable to play on a maximum claim limit retained by the ceding company. Thus no individual claim can by itself excessively unbalance the results.

The excess of claims contract specifies the reinsurer's commitment to each claim (per risk or per event). Generally, this commitment is defined by an intervention limit (deductible) and the extent of the commitment (scope).

- The annual stop-loss treaty: The annual excess loss allows you to work directly on the annual claims charges. As in the case of excess claims, the reinsurer makes a defined commitment beyond a single intervention (the deductible) and a commitment limit (the scope). The difference with excess claims is that the claim expense considered is the total annual claim expense. Another difference is that the limits are generally expressed as a percentage of the gross premiums written by the ceding company.

## 2. Convex function theory

Convex functions are one of the most basic types of functions.

A function f :$R_n \rightarrow$ R is convex if its domain is a convex set and for all x, y in its domain, and all $\lambda \in [0, 1]$, we have :

$$f(\lambda x + (1 - \lambda)y) \leqslant \lambda f(x) + (1 - \lambda)f(y) \tag{1}$$

Convex functions have many applications. It is used to prove some inequalities effortlessly. Also, it has various applications in operation research, Quadratic and Geometric programming problems (Constantin & Persson, 2004, pp. 145-157).

Convex functions are elementary to minimize (for example, any minimum of a convex function is a global minimum). For this reason, there is a wealthy theory for solving convex optimization problems with many practical applications (for example, circuit design, controller design, modelling, etc.)

## 3. De Finetti’sresultats on non-linear problems

Historically, actuaries have focused on optimal reinsurance in a two-dimensional space. Their purpose is to compare and improve their strength and performance. The insurer is always looking to maximize its profit and minimize the standard deviation of this profit (Glineur & Walhin, 2006, pp. 2-8).

Consider a portfolio with n independent risks $X_1, \ldots, X_n$ against a premium $P_1, \ldots, P_n$ . We will protect each risk by proportional reinsurance with a percentage of transfer $(1 - \theta)$. The ceded premium is loaded by a technical load based on the expected value; it is written as follows:

$$P(\text{surrendered}) = (1+\varepsilon_i^{re})(1-\theta)\,E(X) \tag{2}$$

De Finetti suggested choosing a divested quantity $(1- \theta_i)$ that minimizes the variance of the insurer's result and keeps its expectation value constant. The result of the insurer is given by:

$$Z(\theta) = \sum_{i=1}^{n}(P_i - (1+\varepsilon_i^{re})(1-\theta_i)\,E(X_1) - \theta_i X_i) \tag{3}$$

$\Theta$ is a vector of proportionally conserved quantities. $\Theta = (\theta_1, \ldots, \theta_n)$.

The determination of this vector will be a problem of minimizing the volatility of claims:

$$\text{Under the constraints}\quad \begin{cases} \text{Min Var } Z(\theta) \\ E(Z(\theta)) = K \\ \theta_i \geq 0, \quad i = 1, \ldots, n \\ \theta_i \leqslant 1, \quad i = 1, \ldots, n \end{cases}$$

Where K is the expectation value of the result chosen by the insurer.
To solve this problem, it is necessary to understand the variance of claims by the policy.

$$\text{Var}(Z(\theta)) = \text{Var}[-\sum_{i=1}^{n}(\theta_i \text{Var}(X_i)] = \sum_{i=1}^{n}\theta_i^2 \text{Var}(X_i) \tag{4}$$

The expectation value of the result is given by $E(Z(\theta))$

$$\begin{aligned} E(Z(\theta)) &= \sum_{i=1}^{n}(P_i - (1+\varepsilon_i^{re})(1-\theta_i)\,E(X_1) - \theta_i E(X_i)) \\ &= \sum_{i=1}^{n} P_i - \varepsilon_i^{re})(1-\theta_i)\,E(X_1) - \theta_i E(X_i)) \end{aligned} \tag{5}$$

The problem to be solved is, therefore;

$$\text{Min} \sum_{i=1}^{n}\theta_i^2 \text{Var}(X_i)$$

Under the constraints:

$$\begin{cases} \sum_{i=1}^{n} \varepsilon_i^{re}\,(1-\theta_i)\,\mathrm{E}(X_1) = -\mathrm{K} + \sum_{i=1}^{n} E(X_i) \\ \theta_i \geq 0 \quad , \quad i=1,\dots,n \\ \theta_i \leqslant 1 \quad ,i=1,\dots,n \\ -\theta_i \leqslant 0 \quad ,i=1,\dots,n \end{cases}$$

The solution of this modelling is:

$$\theta_i = \min[1, \max(0, 1-\frac{\lambda\mu \mathrm{E}(X_i)}{\mathrm{Var}(X_i)})] \tag{6}$$

λ is the constant of the Lagrange multiplier.

## 4. Optimization steps

### 4.1. The probability of ruin

$$\mathrm{P}(X > R+P) \leqslant \alpha \tag{7}$$

R: the amount of the reserves of the company.
P: the number of premiums.
X: the overall burden of the claim
α: minimum level fixed a priori.

The first goal of determining the probability of ruin is to logically evaluate the wealth of the company (Deelstra & Plantin, 2006), that is to say, the probability that the scenario translating a failure is realized, and to estimate the initial level of reserves to make this probability sufficiently low.

The insurer may consider using reinsurance to minimize the likelihood of bankruptcy. In the case of a quota share, the minimization takes place as well (Azcue, P. &Muler, N. 2005);

We consider a homogeneous group of n risks. The random variable is the expenditure from risk no. I and the total amount of expenses of the group for the current year and therefore:

$$X = Y_1 + Y_2 +, \dots, + Y_n$$

The pure premium is assumed to be charged proportionally to the expected value. For simplicity, it will be assumed that the technical loading rate applied by the reinsurer to the pure reinsurance premium is also equal to λ and that there is no commission payment by reinsurance.

By applying the following assumptions:

**H1**: The random variables $Y_1$, ... $Y_n$ are independent and identically distributed.

Given the above ratings, the cumulative annual amount of claims rated X is:

$$X= Y_1+Y_2+,\dots,+Y_n$$

X follows the binomial law of parameters n and p, which is noted $X$ ~ Ɓ (n, p) from which one deduces the mathematical expectation value and the variance of the random variable X:

$$E(X) = n\ p et Var(X) = n\ p\ q$$

Let us introduce the additional hypothesis (H2) below that will allow us to easily carry out mathematical calculations;

**H2:** It is assumed that the number n of insureds is large enough for the law binomial Ɓ (n, p) can be approximated by the normal distribution:

$$N\ (m,\ \sigma\ 2) \text{ with } (m = n\ p \text{ and } \sigma = \sqrt{npq})$$

This approximation will be possible provided that the product npq is sufficiently large.

Approximately:

$$Xi \sim N\ (n\ p,\ n\ p\ q)$$

The random variable θX follows approximately the law N(θ E (X), V (X)).

$$P\ (ruine) = P\ \{\theta X > R + \theta\Pi(X)\} \quad (8)$$

Defining the reduced centred random variable U by:

$$U= \frac{\theta X-E(\theta X)}{\sigma(\theta(X))} \quad (9)$$

$$\text{P (ruined)} = \text{P } \{U > \frac{R+\theta(1+\lambda)E(X)-\theta E(X)}{\theta\sigma(X)}\} \quad (10)$$

$$\text{P (ruined)} = \text{P } \{U > \frac{R+\theta\lambda E(X)}{\theta\sigma(X)} \} \quad (11)$$

Or again:

$$\text{P (ruined)} = 1- F_0\left( \frac{R}{\theta\sigma(X)}+\lambda\frac{E(X)}{\sigma(X)}\right) \quad (12)$$

Moreover, we have seen previously that the profit before reinsurance is equal to:

$$B= \Pi(X)- X \quad (13)$$

Its mathematical expectation value and its variance have for expression:

$$E(B) = \lambda\ E(X)\sigma(X) = \sigma(B)$$

If the factor of safety before reinsurance is greater than or equal to the value t desired by the insurer (which is judged 4), that is:

$$T= \frac{R+E(B)}{\sigma(X)} = \frac{X+\lambda E(X)}{\sigma(X)} \geq t \quad (14)$$

So, reinsurance is not necessary. Conversely, if the previous inequality is not verified, the insurer will have to resort to reinsurance.

We are now in this case, and it is assumed that the insurer wishes to best adjust the complete conservation for a given risk.

The insurer's annual profit after reinsurance, denoted B (θ), is given by:

$$B(\theta) = \Pi(X) - \theta X - (1 + \mu)\ (1 - \theta)\ E(X) \quad (15)$$

Or the technical bonus can also be written

$$\Pi\ (X) = (1 + \lambda)E(X) \quad (16)$$

Given the above assumptions, we deduce the expectation value and the variance:

$$E(B(\theta)) = (\lambda - \mu)E(X) + \mu\theta E(X) \tag{17}$$

And:

$$Var(B(\theta)) = \theta^2 Var(X) \tag{18}$$

Alternatively, the technical bonus can also be written as:
$\Pi(X) = (1 + \lambda) E(X)$. Given the above assumptions, we deduce the expectation and the variance: $0 \leqslant \theta \leqslant 1$

$$T(\theta) = \frac{R + E(B(\theta))}{\sigma(B(\theta))} = \frac{R + (\lambda - \mu)E(X) + \mu\theta E(X)}{\theta\sigma(X)} \tag{19}$$

As we are in the context of proportional share reinsurance, the insurer commission = reinsurance commission, and we write:

$$T(\theta) = \frac{R + \theta\lambda E(X)}{\theta\sigma(X)} \tag{20}$$

Retention is sought from the retention coefficient T(θ):

$$T(\theta) \geq 4 \Rightarrow \frac{R + \theta\lambda E(X)}{\theta\sigma(X)} \geq 4$$

$$\theta \leqslant \frac{R}{4\sigma(X) - \lambda E(X))} \tag{21}$$

Therefore, the probability of ruin will allow us to determine the insurer's retention interval. For this purpose, we had the idea of combining the probability of ruin theorem with convex optimization to accurately determine the insurer's retention (Hess, 2000, pp. 16-22).

## 4.2. De Finetti's optimization on proportional reinsurance

After defining the retention interval from the probability of ruin, De Finetti's model developed (3) will allow us to determine the percentage that represents the insurer's retention accurately. To further explain, we review De Finetti's results:

$$\theta i = \min\ [1, \max(0, 1 - \frac{\lambda\mu E(X_i)}{Var(X_i)})] \tag{22}$$

These results could be found based on nonlinear optimisation results that generalise Lagrange multipliers under inequality constraints.

## 5. Data application

We used data from the CAARAMA insurance company in Algiers for the digital application. Our work is presented as follows:

### 5.1. The probability of ruin calculation

Applying the probability of ruin to different reinsurance treaties would be difficult, especially for non-proportional treaties that require more statistics (Hull, 2013). We will apply this theorem in a proportional quota treaty for our study. This treaty is the most answered in Algeria, given the lack of experience and control of risk in personal insurance.

To be able to calculate the probability of ruin for this portfolio, it must be verified that the following:

- Claims are independent and identically distributed.
- The which represents the disaster attributed to each police, follow the normal law.

It is already known that the claims are independent and identically distributed, remains to verify their normality:

Normality test results

| |
|---|
| data: Charge de sinistre<br>W = 0.64228, p-value < 0.22 |

Source: Prepared by authors with CARAMMA data elaborated with R3.5.

If the p-value is below the fixed alpha level (often 0.05), then the null hypothesis is rejected, and it is concluded that the disaster distribution is not normally distributed.

The p-value is equal to 0.22 > 0.05, which implies the normality of the amounts of claims.

As a result, we continue:

$$E(X)=\frac{1}{n}\sum X_i = 3\ 363\ 088{,}08$$
$$\sigma(X)=\left(\frac{1}{n}\sum(X_i - E(x))^2\right)^{1/2} = 4\ 178\ 268{,}38$$

For a security load $\lambda$=15%

$$\Pi(X)= (1+\lambda)\ E(X) = 3\ 867\ 551{,}29$$
$$T= 2.51$$

We note that our safety factor is equal to 2.51 < 4 hence the need to resort to reinsurance. In this case, the retention interval is:

$$\theta \leqslant \frac{\mathrm{R}}{4\sigma(\mathrm{X}) - \lambda\mathrm{E}(\mathrm{X}))}$$

$$\theta \leqslant \frac{10000000}{4*4\ 178\ 268{,}38 - 0.15*\ 3\ 363\ 088{,}08}$$

$$\theta \leqslant 61.7\%$$

And so the probability of ruin is:

$$\mathrm{P\ (ruined)} = 1 - F_0\ \left(\frac{\mathrm{R} + \theta\lambda\mathrm{E}(\mathrm{X})}{\theta\sigma(\mathrm{X})}\right)$$

$$= 1 - 0.999968$$

$$\mathrm{P\ (ruined)} = 0.0032\%$$

We can confirm that reinsurance reduces the probability of ruin of the insurance.

After determining the interval of the retention, which corresponds to the commitments of the insurer, one tries to find the percentage which makes it possible to optimize this retention thanks to the modelling of (De Finetti).

## 5.2. Optimization by De Finetti's modelling on a portfolio

De Finetti proposed to analyze proportional reinsurance structures by minimizing the variance of the insurer's gain under the constraint that the expected gain is fixed a priori.

To carry out this optimization process, we assume the following assumptions:

- The overall claim burden over one year is X= $\sum X_i$.
- Each Xi represents the victim attributed to each police. For i = 1, ..., n, the $X_i$ are independent and identically distributed.
- On average, the pure premium used to cover the loss ratio is worth P=E(X).
- The insurer applies an equal load to the equal pure premium for each policy. This shipment is supposed to represent the profit of the insurer. We do not include in this percentage management and acquisition fees, as well as any taxes.
- Therefore, the technical premium that can be used to pay the claims is: P=(1+ $\varepsilon_i^{re}$)E(X).

- Optimal retention refers to θ.

We chose to solve this problem through the R software. The latter allowed us to find an optimal value (or vector) (maximum or minimum) for a formula that is the objective to be achieved under constraints or limits applied to the values or vectors sought, and this in order to solve linear or nonlinear problems, but it is limited to a reduced number of constraints. In the case of our problem, the size of our sample is very large, and as the number of constraints follows the number of variables, we face a significant problem. However, we can always find a workable solution for the reduced problem while respecting the constraints.

The sub-portfolio we have selected includes the largest clients, representing 20% of the initial portfolio.

Table 1. Premium and claims data sub-portfolio (DZD)

| insured | Pi | Xi |
|---|---|---|
| 1 | 18 874 721,62 | 24 427 493,71 |
| 2 | 625 158,00 | 4 141 666,66 |
| 3 | 1 524 649,50 | 6 793 190,00 |
| 4 | 5 968 415,15 | 2 586 010,77 |
| 5 | 2 042 545,64 | 27 681 666,66 |
| 6 | 1 321 601,45 | 1 800 000,00 |
| 7 | 38 758 259,58 | 941 666,66 |
| 8 | 54 145 614,51 | 2 278 125,00 |
| 9 | 28 293 239,63 | 941 666,66 |
| 10 | 87 852,76 | 600 000,00 |
| 11 | 124 794,00 | 100 000,00 |
| 12 | 9 426 211,76 | 907 257,23 |
| 13 | 686 164,67 | 981 531,44 |
| 14 | 691 900,00 | 516 000,00 |
| 15 | 344 824,64 | 2 051 648,75 |
| ∑ | 162 5 952,92 | 76747 923,54 |

Source: insurance company CAARAMA data prepared by Authors on R3.5.

It should be noted that this sub-portfolio is relatively homogeneous insofar as premiums collected by the company sometimes cover the amounts of claims and make a profit, and other times they take losses. However, the turnover is still positive. However, the loss ratio is still important and, therefore, will possibly affect the insurer's retention.

To determine the average profit, we used the result found by the ruin probability method: calculate E (Z (61.7%)).

E(Z(61.7%))= 162 915 952.92-(0.15*0.617*10 603 839.13)-10 603 839.13

E(Z(61.7%))= 151 702 923,2 DA

We can conclude that this amount represents the maximum expected profit that an insurer could expect. Given its reserves which have limited the retention level to 61.7%, the insurer is not aiming for more than its expected profit. Although it can increase its real profit if it keeps more than 61.7%, it does not have the necessary reserves which would allow it, in the event of the occurrence of disasters, to assume its retention.

We can also calculate the minimum expected profit to make it vary between the two terminals:

E(Z(0%))= 152 368 128,8 DA

On R 3.5, we have varied the amount of profit expected to observe, for each amount, the level of retention that minimizes the volatility of claims using the LP function. Our results are as follows:

Table 2. Retention results

| The expected gain | The reinsurer loading | The retention | Min σ(X) |
|---|---|---|---|
| 153 217 000,00 | 15% | 61,70% | 2 643 006,26 |
| 153 150 000,00 | 15% | 56.83% | 2 242 253,10 |
| 153 050 000.00 | 15% | 49.56% | 1 705 372,60 |
| 152 950 000,00 | 15% | 42.29% | 1 241 847,81 |
| 152 890 000,00 | 15% | 37.93% | 998 944,21 |
| 152 800 000,00 | 15% | 31.39% | 648 105,44 |

Source: Prepared by authors using CAARAMA's data on R3.5.

For each expected return value, the R application has found a level of retention considered optimal because it minimizes the loss volatility.

The more the expected profit increases, the more insurance retention will be necessary, but this is not reassuring because the volatility of the accident increases with the expected return value. Indeed, a company wishing to improve the profitability of its portfolio must accept to bear more risks.

The volatility of this portfolio is not significant compared to the expected profit. Here is a portfolio that contains reasonable risks. However, it is essential to consider the level of retention that minimizes the variance in earnings.

The volatility of the claim provides a good indication of the risk of loss. Indeed, when a company decides to increase its retention level to aim for a more attractive profit, it must expect greater loss volatility that could hit its portfolio and fluctuate its results.

The company CAARAMA insurance has opted for retention of 50% applied to this portfolio. Given the lack of data, we can assume two cases:

- The expected profit was 153,050,000 DA;
- Or the negotiations with its reinsurers agreed on 50% as retention.

This retention level can be considered optimal because it reduces the volatility of the claim, but only if the expected profit is equal to 153,050,000 DA.

Determining the optimal retention level for this portfolio depends on the insurer's policy and its attitude to risk (which is measured by the volatility of the claim). The insurance company must target a goal somewhat realistic that corresponds to its target of the year and optimize it (Deelstra & Plantin, 2006).

The optimal retention level will not necessarily be implemented in the portfolio concerned. It will also depend on negotiations with the reinsurer, optimizing its retention.

## 5. Conclusion

Nonlinear optimization provides an essential decision-making tool in quantitative finance, and our work is a critical illustration of the importance of convex function studies in decision-making.

As for the insurer, the principle of risk-sharing between him and the reinsurer is based on actuarial methods that determine the commitment that corresponds to each party. For the retention to be optimal, it must correspond to the strategies predefined by the insurer according to the expected return torque and loss volatility.

The optimization process can be achieved by combining two methods that determine the retention interval and optimize it for a fixed expected benefit.

The theory of ruin helped us to:

- Find the interval of our retention, which should be less than 61.7%;
- To note the positive effect of reinsurance on the insurer's security by minimizing its probability of ruin;
- To calculate the maximum expected profit of the insurer, which was subsequently used in the (De Finetti) method.

Lastly, applying the (De Finetti) method allowed us to determine the exact value of the retention, which contributes to the profit desired by the insurer, and for which the volatility of the claims is minimal. However, this value is not necessarily put into practice; everything depends on the negotiations with the reinsurer and the insurer's attitude against the risk.

We can propose as an extension for our study realistic modelling of the expected profit value.

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